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Scale invariant finger length distribution in viscous fingering patterns

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Abstract : We have studied viscous fingering patterns produced by air displacing a highly viscous oil-paint in a lifting Hele-Shaw cell The Hele-Shaw cell consists of two parallel glass plates, with the more viscous fluid sandwiched in between The plates are separated by a pneumatic cylinder arrangement, when the less viscous fluid enters at the sides to produce the pattern. In the present work we show that the displaced fluid and the displacing fluid both show a distinctive scale invariance. The displaced oil-paint forms a fractal tree-like pattern, whereas the displacing air forms fingers with a scale invariant distribution of lengths The length distribution shows log-periodic oscillations characteristic of discrete scale invariance (DSI).

Keywords : Pattern formation, viscous fingering, scale invariance.

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1. Introduction

Many natural processes such as dendritic growth, growth of microbial colonies, dielectric breakdown and cloud formations show patterns displaying scale invariance in some form [1]. Viscous fingering (VF) is such a process where instability of the interface between two fluids leads to formation of interesting patterns, often fractal.

The Hele-Shaw cell [2] is a simple experimental arrangement where VF patterns can be conveniently produced and studied. The conventional Hele-Shaw cell consists of two parallel glass plates with a viscous fluid (2) sandwiched between them. A less viscous fluid (1) is forced in through a hole at the center of the upper plate. Fluid 1 spreads out in the form of a highly branched pattern which may be fractal.

In the present work, we are concerned with a variation of the Hele-Shaw cell, the lifting Hele-Shaw cell (LHSC) [3,4] ; here the fluid 1 enters at the *sides* when the upper plate is lifted slowly. A typical VF pattern is formed which has an additional interesting feature. Here, the treelike fractal pattern is formed as the background as the air fingers push the fluid 2 away. In the conventional HS cell, the displacing fluid itself forms the fractal pattern. Moreover, in our LHSC pattern the air fingers also exhibit a different type of scale invariance. The finger length distribution of the air fingers shows discrete scale invariance with log-periodic oscillations when $\ln N(y)$ is plotted against $\ln(y)$ [5]. Here, $N(y)$ is the number of fingers with length greater than or equal to y. So the displaced fluid pattern and the complimentary intruding fluid pattern both show their own typical scale invariant character.

Log-periodic oscillations are a characteristic feature of discrete scale invariance (DSI), which has recently been shown to exist in many natural systems [6]. Natural fractals are expected to be random fractals, with no definite factor characterising the scale-invariance, as is found in the deterministic fractals e.g. the Koch curve or Sierpinski gasket. Sornette [6] shows that many natural systems do in fact have such a factor, which is another characteristic of the system, in addition to the Hausdorff dimension. This is manifested in periodic oscillations on the usual log-log plot, whose slope gives the Hausdorff dimension.

2. Experimental arrangement

The LHSC consists of two circular glass plates of about 10 cm diameter. The upper plate can be raised or lowered by a pneumatic cylinder arrangement. The two plates are always parallel. To generate the pattern, a blob of oil-paint is placed at the centre of the lower plate, and the upper plate is brought to press down on it. The paint spreads out with a more or less circular outline. Now the upper plate is raised slowly and air enters the gap, displacing the paint and forms the characteristic pattern shown in Figure 1.

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3. Results

In Figure 1, the white lines are the highly viscous fluid 2 (here oil-paint). In the manually separated LHSC, the pattern is much more branched and the paint pattern is a fractal with hausdorff dimension close to 1.5, as reported in previous work [4]. In the mechanically driven arrangement used in the present work, smaller patterns of size close to 1 cm have very little branching and the fractal nature is not so evident.

Figure 1. A typical pattern produced by air displacing oil-paint in the LHSC

The dark gaps between the white lines are the air fingers. In this paper, we are concerned with the air fingers. We have measured the lengths of the air fingers and determined the cumulative distribution N(y). In N(y) plotted against ln(y) is

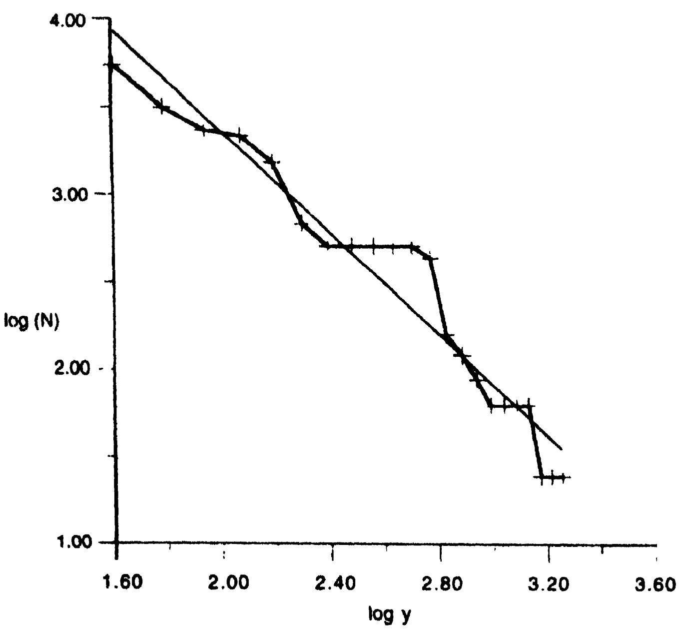

Figure 2. Log N(y) plotted against log(y) shows an average linear behaviour with periodic oscillations.

shown in Figure 2 (here ln is the natural logarithm). On the average, the variation is linear with a slope of – 1.4. This indicates a power law behaviour

$$N(y) = const.\, y^{-m}\,, \qquad (1)$$

with m = 1.4, approximately. However, roughly periodic oscillations are clearly seen decorating the average straight line. We have taken the derivative of the curve shown in Figure 2 to bring out the oscillations more clearly. It is shown in Figure 3. A

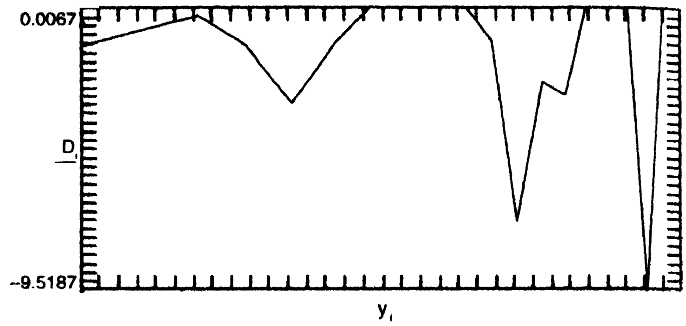

Figure 3. The derivative of the curve in Figure 2, dlog N(y)/d log y vs log(y) shows the oscillations more prominently

lomb periodogram of the derivative gives the most prominent period as close to 1.5, this is shown in Figure 4. If f denotes the frequency of the log- periodic oscillation, the scaling factor for invariance b is given by

$$f = 1/\log(b)\,. \qquad (2)$$

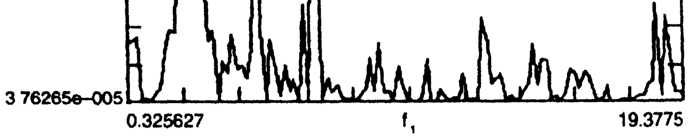

Figure 4. The lomb periodogram of Figure 3 gives the frequencies of the oscillations in Figure 2 and 3.

4. Conclusion

We have shown that viscous fingering belongs to the class of natural phenomena which exhibit DSI. The period of the DSI is another important characteristic of the process [6] in addition to the hausdorff dimension. It indicates that there is a hidden determinism in the apparently stochastic process and the value of this period may help in specifying the formation mechanism of the pattern. This is seen for example in the analysis of the length distribution of cracks in a geological formation [7].

The presence of scale invariance in the air fingers as well as the complimentary paint pattern, is very interesting. The mass distribution of the compliment of a fractal pattern must of course, be compact.

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